1-1 Operations on Real Numbers

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I CAN… reason about operations on real numbers.

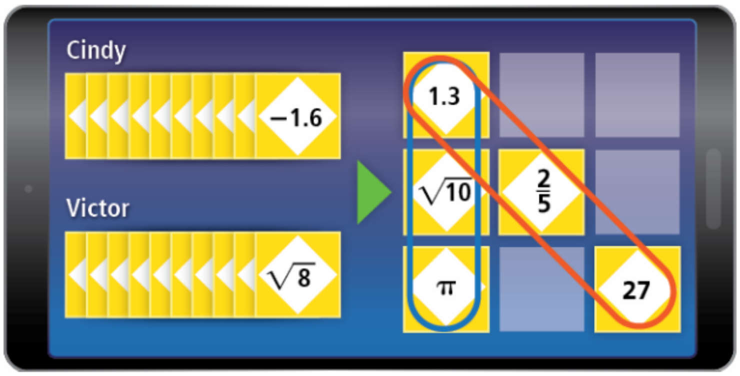
VOCABULARY

- element of a set
- set
- subset

Activity Assess

CRITIQUE & EXPLAIN

Cindy and Victor are playing a math game. The winner must get three in a row of the same type of real number and justify how the numbers are alike. Cindy said she won because she was able to get three rational numbers on a diagonal. Victor said he won with three positive numbers in a column.



A. Can both players say they won for different reasons? Explain.

B. **Reason** Can you make other groups using the numbers shown that are all the same kind of real number? In how many ways can you do this?

ESSENTIAL QUESTION

How can you classify the results of operations on real numbers?

EXAMPLE 1 Understand Sets and Subsets

In the set of numbers from 1 to 10, which elements are in both the subset of even numbers, and the subset of multiples of 5?

A **set** is a collection of objects such as numbers. An **element of a set** is an object that is in the set. Write a set by listing the elements, enclosed in curly braces ("{"and "}").

Name of the set

$$A = \underbrace{\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}}_{\text{Elements of the set}}$$

MAKE SENSE AND PERSEVERE Write out each subset. Then see which elements are common to both.

Set B is a **subset** of set A if each element of B is also an element of A.

$B = \{2, 4, 6, 8, 10\}$ Elements of A that are even

$C = \{5, 10\}$ Elements of A that are multiples of 5

The number 10 is the only number that is an element of both subsets.

Try It! **1.** Which numbers in set A are elements in both the subset of odd numbers and the subset of multiples of 3?

APPLICATION

EXAMPLE 2 Compare and Order Real Numbers

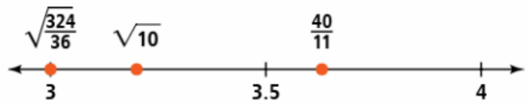
Jim is playing a math game where he needs to put a set of three cards in numerical order. His cards show $\frac{40}{11}$, $\sqrt{\frac{324}{36}}$, and $\sqrt{10}$. Order the cards from least to greatest.

STUDY TIP
It is easier to compare and order real numbers when they are all in the same form. Rewrite real numbers to the equivalent decimal form so you can compare them easily.

Find the decimal equivalent for each number.

$$\frac{40}{11} = 3.\overline{63} \qquad \sqrt{\frac{324}{36}} = \frac{18}{6} = 3 \qquad \sqrt{10} \approx 3.2$$

Plot the numbers on a number line.



From least to greatest, the order of the cards is $\sqrt{\frac{324}{36}}$, $\sqrt{10}$, and $\frac{40}{11}$.

Try It! 2. Order each set of cards from least to greatest.

a. 0.25, $\sqrt{\frac{1}{9}}$, $\frac{6}{25}$

b. $\sqrt{\frac{121}{25}}$, 2.25, $\sqrt{5}$

CONCEPTUAL UNDERSTANDING

EXAMPLE 3 Operations With Rational Numbers

A. Is the sum of two rational numbers always a rational number?

You can try several different cases of adding two rational numbers.

$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6} \qquad \frac{7}{8} + \frac{3}{4} = \frac{13}{8} \qquad \frac{11}{5} + \frac{1}{6} = \frac{71}{30}$$

In each case, the sum is also rational. But you cannot try *every* pair of rational numbers since there are infinitely many of them. How can you know whether it is true for *all* rational numbers?

Use variables to represent any rational number.

a, b, c, and d are integers with $b \neq 0$, and $d \neq 0$.

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd}$$
$$= \frac{ad + bc}{bd}$$

Since $b \neq 0$ and $d \neq 0$, $bd \neq 0$ also.

Since $ad + bc$ and bd are integers, and $bd \neq 0$, the sum is rational.

B. Is the product of two rational numbers always a rational number?

Use the same strategy as in part A, using variables to represent any rational number.

a, b, c, and d are integers with $b \neq 0$, and $d \neq 0$.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Since $b \neq 0$ and $d \neq 0$, $bd \neq 0$ also.

Since ac and bd are integers, and $bd \neq 0$, the product is rational.

Try It! 3. Is the quotient of two rational numbers always a rational number? Explain.

EXAMPLE 4 Operations With Rational and Irrational Numbers

A. Is the sum of a rational number and an irrational number rational or irrational?

If you could write the sum of an irrational and a rational number as a rational number, you could write the following equation.

irrational

rational $\frac{a}{b} + c = \frac{p}{q}$ rational

$$c = \frac{pb - aq}{bq}$$

In the rational numbers above, *a, b, p,* and *q* are integers, with $b \neq 0$ and $q \neq 0$. This means that

- $pb - aq$ is an integer and
- bq is an integer not equal to 0.

Therefore $\frac{pb - aq}{bq}$ is a rational number. But this is equal to *c*, an irrational number. Can a number be both rational and irrational? No, it cannot.

So what went wrong? The mistake was to assume that you could write the sum $\frac{a}{b} + c$ in the form $\frac{p}{q}$.

The sum of a rational number and an irrational number is always an irrational number.

B. Is the product of a rational number and an irrational number rational or irrational?

Write the product as a rational number.

irrational

rational $\frac{a}{b} \cdot c = \frac{p}{q}$ rational

$$c = \frac{bp}{aq}$$

As in part A, *c* is both rational and irrational. So the assumption that you can write $\frac{a}{b} \cdot c$ as $\frac{p}{q}$ at all is wrong.

Also notice that in order to divide by *a* when calculating *c*, you have to assume that $a \neq 0$. What happens in the original equation if $a = 0$?

Then $\frac{a}{b} = 0$, and $\frac{a}{b} \cdot c = 0$ for *any* number *c*.

So the product of a rational number and an irrational number is always irrational, unless the rational number in the product is 0.

COMMON ERROR

If you do not address the case where $a = 0$, you might conclude that the product of any rational number and any irrational number is irrational. But that is not true.

Try It! **4.** Is the difference of a rational number and an irrational number always irrational? Explain.

Concept Summary Assess

CONCEPT SUMMARY Operations on Real Numbers

WORDS	The sum of two rational numbers is always rational. The product of two rational numbers is always rational.	The sum of a rational number and an irrational number is always irrational. The product of a nonzero rational number and an irrational number is always irrational.
NUMBERS	Sums: $\frac{2}{9} + \frac{4}{6} = \frac{32}{36}$ Products: $\frac{2}{9} \cdot \frac{4}{6} = \frac{8}{54}$	Sums: $\sqrt{3} + \frac{1}{3} = \frac{3\sqrt{3} + 1}{3}$ Products: $\sqrt{3} \cdot \frac{1}{3} = \frac{\sqrt{3}}{3}$
ALGEBRA	Sums: $\frac{a}{b} + \frac{c}{d} = \frac{ad + cb}{bd}$ Products: $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$	Sums: $\frac{a}{b} + c \neq \frac{p}{q}$, when c is irrational Products: $\frac{a}{b} \cdot c \neq \frac{p}{q}$, when c is irrational

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How can you classify the results of operations on real numbers?

2. **Communicate Precisely** Explain why the sum of a rational number and an irrational number is always irrational.

3. **Vocabulary** Are the rational numbers a *subset* of the *set* of all real numbers? Are the rational numbers a *subset* of the irrational numbers? Explain?

4. **Error Analysis** Jacinta says that the product of a rational number and an irrational number is always irrational. Explain her error.

5. **Reason** Let $D = \{-2, -1, 0, 1, 2\}$. Is D a subset of itself? Explain.

Do You KNOW HOW?

Determine whether set B is a subset of set A.

6. $A = \{0, 1, 2, 3, 4\}$
   $B = \{1, 2\}$

7. $A = \{2, 3, 5, 7, 11\}$
   $B = \{3, 5, 7, 9, 11\}$

Order each set of numbers from least to greatest.

8. $\sqrt{200}$, 14, $\frac{41}{3}$

9. $\frac{2}{3}$, $\sqrt{\frac{9}{16}}$, 0.6

10. The park shown is in the shape of a square. Is the perimeter rational or irrational?

PRACTICE & PROBLEM SOLVING

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UNDERSTAND

11. **Reason** Identify each solution as rational or irrational.

a. $\frac{4}{7} + \frac{-1}{3}$ **b.** $\sqrt{4} \cdot \frac{2}{5}$

12. **Higher Order Thinking** Is the product of two irrational numbers always an irrational number? Explain.

13. **Error Analysis** Describe and correct the error a student made when ordering numbers from least to greatest.

$\sqrt{144}, \frac{234}{3}, 68.12$

$\sqrt{144} = 72$

$\frac{234}{3} = 78$

$68.12, \sqrt{144}, \frac{234}{3}$ ✗

14. **Mathematical Connections** The bulletin board is in the shape of a square. Find two rational numbers that are within $\frac{1}{8}$ in. of the actual side length.

15. **Construct Arguments** Tell whether each statement is *always true, sometimes true,* or *never true*. Explain.

a. An integer is a whole number.

b. A natural number is a rational number.

c. An irrational number is an integer.

PRACTICE

List all subsets of the real numbers from the list below that each number belongs to. SEE EXAMPLE 1

- real numbers
- irrational numbers
- rational numbers
- integers
- whole numbers

16. 10.5 **17.** $\frac{4}{7}$

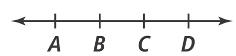
18. 6 **19.** 0

20. $\sqrt{2}$ **21.** –29

Order the numbers shown from least to greatest. SEE EXAMPLE 2

22. $3.5, \frac{10}{3}, \sqrt{14}$ **23.** $\frac{1}{3}, 0.1\overline{6}, \sqrt{\frac{1}{4}}$

Match each number to the letter that represents its position on the number line. SEE EXAMPLE 2



24. $-\sqrt{120}$

25. $-\sqrt{\frac{400}{4}}$

26. $-\frac{23}{2}$

27. –11.75

Determine whether each sum, difference, product, or quotient represents a rational number or an irrational number. Explain how you know without simplifying. SEE EXAMPLES 3 AND 4

28. $\frac{6}{23} - \frac{\sqrt{2}}{2}$

29. $\frac{6}{23} - \frac{15}{127}$

30. $\frac{6}{23} \div \frac{15}{127}$

31. $\frac{6}{23} \div \frac{\sqrt{2}}{2}$

32. Is the difference of two rational numbers always a rational number? Explain. SEE EXAMPLE 3

33. Is the quotient of a rational number and an irrational number always irrational? Explain. SEE EXAMPLE 4

PRACTICE & PROBLEM SOLVING

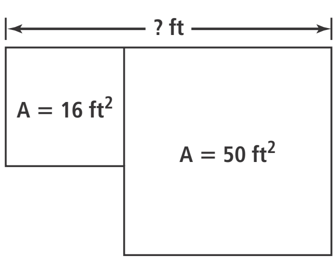
APPLY

34. **Make Sense and Persevere** Adam wraps the top edge of the gift box shown with gold ribbon.

The top and bottom edges of the box are square. If Adam has $24\frac{1}{4}$ in. of gold ribbon, does he have enough to decorate the top of box?

35. **Reason** In statistics, *continuous data* can have values equal to any real number, such as the average temperature for an area or the number of inches of rainfall. Other sets of data are *discrete*. Examples of discrete data are the number of students in a school district, the number of home runs hit by a baseball team in a season, and the number of letters handled by the post office each month. Which subset of the real numbers is the best one to use to describe discrete data?

36. **Make Sense and Persevere** Helena builds a shed in her backyard. There is a larger section for large tools, like her lawn mower, and a smaller section for small tools. What is the length of the entire shed? What type of number is the length? List as many types of numbers for the length as you can.



ASSESSMENT PRACTICE

37. Is 0.62473 a member of the set? For each set of real numbers, select *Yes* or *No*.

	Yes	No
natural numbers	❏	❏
whole numbers	❏	❏
integers	❏	❏
rational numbers	❏	❏
irrational numbers	❏	❏
real numbers	❏	❏

38. **SAT/ACT** What is the square root of $\sqrt{\frac{144}{256}}$?

Ⓐ $\frac{2}{3}$ Ⓑ $\frac{3}{4}$ Ⓒ $\frac{3}{16}$ Ⓓ $\frac{9}{4}$ Ⓔ $\frac{9}{16}$

39. **Performance Task** A basketball coach is considering three players for Most Valuable Player (MVP). The table shows the proportion of shots each player made of the shots they attempted.

Player	Free Throws	Field Goals (2 pts)	3-Point Shots
Martin	71%	49.5%	32%
Corey	$\frac{4}{5}$	$\frac{9}{20}$	$\frac{1}{3}$
Kimberly	0.857	0.448	0.338

Part A For a technical foul, the team can pick any player they want to shoot the free throw. Which player should the team pick? Explain.

Part B Which player is most successful with their field goal shots? Explain.

Part C Rank the players by the percentage of the 3-point shots each made.

Part D If all the players attempted the same number of shots, which player would you choose as the MVP? Justify your answer.

11. **a.** rational

b. rational

13. The student should have written $\sqrt{144} = 12$. So, the correct order is $\sqrt{144}$, 68.12, $\frac{234}{3}$.

15. **a.** sometimes true; Positive integers and zero are whole numbers, but negative integers are not.

b. always true; Natural numbers are a subset of rational numbers.

c. never true; Although integers and irrational numbers are both subsets of real numbers, they have no elements in common.

17. rational, real

19. whole, integers, rational, real

21. integers, rational, real

23. 0.16, $\frac{1}{3}$, $\sqrt{\frac{1}{4}}$

25. *D*

27. *A*

29. Rational; can be expressed as a ratio.

31. Irrational; divisor is irrational.

33. Yes; the quotient of a rational number and an irrational number can be rewritten as a ratio with a rational number in the numerator and an irrational number in the denominator, so the quotient will always be irrational.

35. rational numbers

37. No; No; No; Yes; No; Yes

39. **Part A** Kimberly; She has the highest free throw percentage.

Part B Martin; 49.5% is greater than $\frac{9}{20}$, or 45% and 0.448, or 44.8%

Part C Kimberly, Corey, Martin

Part D Kimberly; Kimberly makes about 54.8% of her shots, while Corey only makes 52.8%, and Martin only makes 50.8%.